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Einstein–Podolsky–Rosen pair states and the charge-amplitude representation for complex scalar fields

Hong-yi Fan†‡ and Zeng-bing Chen‡§

† CCAST (World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China
‡ Department of Material Science and Engineering, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

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Abstract. For complex scalar fields $\phi(x)$ and $\phi^\dagger(x)$, we introduce the entangled states $\|\xi\rangle$ which are their common eigenvectors. The $\|\xi\rangle$ states possessing complete and orthonormal properties are a field-theoretical generalization of the Einstein–Podolsky–Rosen pair states in quantum mechanics. Based on the new $\langle\xi\|$ representation, we obtain the complete and orthonormal states $\|\{q\}, r\rangle$, the common eigenvectors of the charge operator and $\phi^\dagger(x)\,\phi(x)$, with which the charge lowering and raising operators can be naturally defined. The applications of our formulation are briefly discussed.

The concept of entanglement has played a key role in some fundamental problems in quantum mechanics [1–3]. In an entangled quantum state, a measurement performed on one part of the system provides information on the remaining part, as first pointed out by Einstein, Podolsky and Rosen (EPR) [4]. Thus, entanglement, though EPR found it to be unbelievable, is essential, for example, in quantum teleportation [5] and has now been known as a basic feature of quantum mechanics. In EPR's original argument, they used the fact that the relative position $Q_1 - Q_2$ and the total momentum $P_1 + P_2$ of two particles are permutable operators and therefore have common eigenstates which should be entangled in the EPR sense. Remarkably, the common eigenstates $|\eta\rangle$ of compatible operators $(Q_1 - Q_2, P_1 + P_2)$ and the common eigenstates $|\xi\rangle$ of compatible operators $(Q_1 + Q_2, P_1 - P_2)$ were explicitly constructed in two-mode Fock space recently [6]. The construction of the EPR pair states, $|\eta\rangle$ and $|\xi\rangle$, motivates us to seek their field-theoretical generalization.

Our purpose in this paper is twofold. (a) We first reveal that the entanglement involved in the EPR pair states also exists in the context of the quantum theory of complex scalar fields (CSF). We will construct the explicit entangled eigenstates $\|\xi\rangle$, which are a field-theoretical generalization of $|\xi\rangle$, of the CSF $\phi(x)$ and $\phi^\dagger(x)$ in the Fock space. (b) We then show that from $\|\xi\rangle$, one can project out states $\|\{q\}, r\rangle$ of definite charge. This procedure is similar to that used in obtaining the conserved-charge coherent states from two-mode canonical coherent states [7]. The complete and orthonormal states $\|\{q\}, r\rangle$ are shown to be the common eigenvectors of the charge operator and $\phi^\dagger(x)\,\phi(x)$, with which the charge lowering and raising operators can be naturally defined. The applications of this paper are also briefly addressed.

In the CSF theory, ϕ and $\phi^\dagger$ are two independent fields. The CSF can be canonically quantized and split into a positive- and negative-frequency part as [8]

$$\phi(\boldsymbol{x}, t) = \phi_+ + \phi_- \qquad \phi^\dagger(\boldsymbol{x}, t) = \phi_+^\dagger + \phi_-^\dagger \tag{1}$$

§ Author to whom correspondence should be addressed at Hefei.

where

$$\phi_+ = \sum_p a_p f_p(\boldsymbol{x}, t) = (\phi_-^\dagger)^\dagger \qquad \phi_- = \sum_p b_p^\dagger f_p^*(\boldsymbol{x}, t) = (\phi_+^\dagger)^\dagger \tag{2}$$

with $[a_p, a_{p'}^\dagger] = [b_p, b_{p'}^\dagger] = \delta_{pp'}$, $f_p(\boldsymbol{x}, t) = (2V\omega_p)^{-1/2}\,\mathrm{e}^{\mathrm{i}(\boldsymbol{p}\cdot\boldsymbol{x}-\omega_p t)}$, $\omega_p = \sqrt{m^2+\boldsymbol{p}^2}$ (m is the mass of the field quanta) and V is the normalization volume. Consequently,

$$\begin{aligned}\left[\phi_+^\dagger(\boldsymbol{x}, t), \phi_-(\boldsymbol{x}', t)\right] &= \left[\phi_+(\boldsymbol{x}, t), \phi_-^\dagger(\boldsymbol{x}', t)\right] \\ &= \sum_p \frac{\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{x}')}}{2V\omega_p} \equiv \frac{1}{2}G(\boldsymbol{x}-\boldsymbol{x}').\end{aligned} \tag{3}$$

The inverse of $G(\boldsymbol{x}-\boldsymbol{x}')$ is $G^{-1}(\boldsymbol{x}-\boldsymbol{x}') = V^{-1}\sum_p \omega_p\,\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{x}')}$, as it satisfies

$$\int \mathrm{d}^3y\, G(\boldsymbol{x}-\boldsymbol{y})G^{-1}(\boldsymbol{y}-\boldsymbol{x}') = \frac{1}{V}\sum_p \mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot(\boldsymbol{x}-\boldsymbol{x}')} = \delta(\boldsymbol{x}-\boldsymbol{x}'). \tag{4}$$

The conjugate fields $\Pi(\boldsymbol{x}, t) \equiv \partial_t\phi^\dagger(\boldsymbol{x}, t)$ and $\Pi^\dagger(\boldsymbol{x}, t) \equiv \partial_t\phi(\boldsymbol{x}, t)$. The non-vanishing equal-time commutation relations are $[\phi(\boldsymbol{x}, t), \Pi(\boldsymbol{x}', t)] = \mathrm{i}\delta(\boldsymbol{x}-\boldsymbol{x}')$ and $[\phi^\dagger(\boldsymbol{x}, t), \Pi^\dagger(\boldsymbol{x}', t)] = \mathrm{i}\delta(\boldsymbol{x}-\boldsymbol{x}')$.

Since $\phi(x)$ and $\phi^\dagger(x)$ are commutative and independent of each other, we can construct their common eigenstates which in the Fock space are given by (from now on we work in the Schrödinger picture, for example, $\phi(x) \equiv \phi(\boldsymbol{x}, t=0)$)

$$\begin{aligned}\|\xi\rangle = \left[\det\left(\tfrac{1}{2}G\right)\right]^{-1/2} \exp\bigg\{\iint \mathrm{d}^3x\,\mathrm{d}^3x'\, G^{-1}(\boldsymbol{x}-\boldsymbol{x}')\big[-\xi^*(\boldsymbol{x})\xi(\boldsymbol{x}') + 2\xi(\boldsymbol{x})\,\phi_-^\dagger(x') \\ +2\xi^*(\boldsymbol{x})\,\phi_-(x') - 2\phi_-^\dagger(x)\,\phi_-(x')\big]\bigg\}\|00\rangle\end{aligned} \tag{5}$$

where the vacuum state of the CSF is annihilated by both a_p and b_{-p}: $a_p|00\rangle_p = b_{-p}|00\rangle_p = 0$ and $\|00\rangle = \prod_p |00\rangle_p$. In fact, by acting $\phi_+(x)$ $[\phi_+^\dagger(x)]$ on $\|\xi\rangle$ and using equations (3) and (4), we are led to

$$\begin{aligned}\phi_+(x)\|\xi\rangle &= [\xi(\boldsymbol{x}) - \phi_-(x)]\|\xi\rangle \\ \phi_+^\dagger(x)\|\xi\rangle &= \left[\xi^*(\boldsymbol{x}) - \phi_-^\dagger(x)\right]\|\xi\rangle.\end{aligned} \tag{6}$$

Therefore, the $\|\xi\rangle$ states are indeed the common eigenvectors of ϕ and $\phi^\dagger$. The $\|\xi\rangle$ state can also be written in the momentum space. Using equation (2) and the Fourier transformation $\xi(\boldsymbol{x}) = V^{-1/2}\sum_p \xi_p\,\mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{x}}$, we obtain

$$\|\xi\rangle = \prod_p \sqrt{2\omega_p}\exp\left[-\omega_p|\xi_p|^2 + \sqrt{2\omega_p}\,\xi_p a_p^\dagger + \sqrt{2\omega_p}\,\xi_p^* b_{-p}^\dagger - a_p^\dagger b_{-p}^\dagger\right]|00\rangle_p \equiv \prod_p |\xi_p\rangle. \tag{7}$$

The simultaneous appearance of both $a_p^\dagger$ and $b_{-p}^\dagger$ originates, of course, from the conservation of momentum.

It is obvious that each $\boldsymbol{p}$-mode component of $\|\xi\rangle$ in equation (7) is equivalent to the entangled states $|\xi\rangle$ in the two-mode case [6]. Therefore, the newly constructed eigenstates $\|\xi\rangle$ are a field-theoretical generalization of $|\xi\rangle$, and the entanglement involved in the former occurs between the positively and negatively charged quanta. In this sense we call $\|\xi\rangle$ the entangled eigenstates (the 'EPR pair states') of the CSF. The eigenstates $\|\xi\rangle$ are also entangled in the sense that they are not a direct product of eigenstates of two real components, $\phi_1 = (\phi+\phi^\dagger)/\sqrt{2}$ and $\phi_2 = \mathrm{i}(\phi^\dagger - \phi)/\sqrt{2}$, of the CSF. We emphasize that the entangled eigenstates in equation (5) or (7) are in accordance with the superselection rule [9]. The $\|\xi\rangle$ states comprise both $a_p^\dagger$

and $b^\dagger_{-p}$ acting on the vacuum state in equation (7). They create the positive and negative quanta simultaneously from the vacuum, and this ensures the conservation law of charge. The common eigenstates of Π and $\Pi^\dagger$ can be similarly constructed and are a field-theoretical generalization of $|\eta\rangle$ [10].

We can prove the complete and orthonormal properties of $\|\xi\rangle$ as follows. From the well known expression $\|00\rangle\langle 00\| = \prod_p : \mathrm{e}^{-a_p^\dagger a_p - b_{-p}^\dagger b_{-p}} :$, where : : denotes normal ordering, and the inverse transformations of equation (2), we obtain the normal product forms of the vacuum state projector $\|00\rangle\langle 00\|$ in terms of the field variables:

$$\|00\rangle\langle 00\| =: \exp\left\{-2\iint \mathrm{d}^3x\,\mathrm{d}^3x'\,G^{-1}(\boldsymbol{x}-\boldsymbol{x}')\big[\phi_-^\dagger(x)\,\phi_+(x') + \phi_-(x)\,\phi_+^\dagger(x')\big]\right\} : . \tag{8}$$

Using equations (5) and (8), we can prove the completeness relation of $\|\xi\rangle$ as

$$\left[\frac{\mathrm{d}^2\xi}{\pi}\right]\|\xi\rangle\langle\xi\| = \big[\det\big(\tfrac{1}{2}G\big)\big]^{-1} : \int\left[\frac{\mathrm{d}^2\xi}{\pi}\right]\exp\bigg[-2\iint \mathrm{d}^3x\,\mathrm{d}^3x'\,G^{-1}(\boldsymbol{x}-\boldsymbol{x}') \times[\xi(\boldsymbol{x})-\phi(x)][\xi^*(\boldsymbol{x}')-\phi^\dagger(x')]\bigg] := 1. \tag{9}$$

Here the integrals are of course the functional ones [8, 11]. In arriving at the completeness relation, we have used the technique of integration within an ordered product (IWOP) of operators [12].

The Hermitian conjugates of equation (6) are $\langle\xi\|\phi^\dagger(x) = \xi^*(\boldsymbol{x})\langle\xi\|$ and $\langle\xi\|\phi(x) = \xi(\boldsymbol{x})\langle\xi\|$. Thus $\langle\xi'\|\phi(x)\|\xi\rangle = \xi(\boldsymbol{x})\langle\xi'\|\xi\rangle = \xi'(\boldsymbol{x})\langle\xi'\|\xi\rangle$ and $\langle\xi'\|\phi^\dagger(x)\|\xi\rangle = \xi^*(\boldsymbol{x})\langle\xi'\|\xi\rangle = \xi'^*(\boldsymbol{x})\langle\xi'\|\xi\rangle$. It then follows that the eigenvectors $\|\xi\rangle$ are orthonormal:

$$\langle\xi'\|\xi\rangle = [\pi]\delta^{(2)}[\xi'-\xi] \tag{10}$$

where the functional delta function $\delta^{(2)}[\xi] \equiv \delta[\xi]\delta[\xi^*]$. The $\|\xi\rangle$ set is therefore qualified to be a complete and orthonormal representation, the $\langle\xi\|$ representation. One may also use the IWOP technique to obtain equations (9) and (10) starting from equation (7) since we have

$$\int\frac{\mathrm{d}^2\xi_p}{\pi}|\xi_p\rangle\langle\xi_p| = 1 \qquad \langle\xi_p'|\xi_p\rangle = \pi\delta^{(2)}(\xi_p-\xi_p') \tag{11}$$

similar to the $|\xi\rangle$ states [6]. The $\langle\xi\|$ representation enables us to define the amplitude and phase fields of ϕ, respectively, as

$$\begin{aligned} A(x) &\equiv \sqrt{\phi^\dagger\phi} = \int\left[\frac{\mathrm{d}^2\xi}{\pi}\right]|\xi(\boldsymbol{x})|\|\xi\rangle\langle\xi\| \\ \mathrm{e}^{\mathrm{i}\theta(x)} &\equiv \frac{\phi}{A} = \int\left[\frac{\mathrm{d}^2\xi}{\pi}\right]\mathrm{e}^{\mathrm{i}\arg\xi(x)}\|\xi\rangle\langle\xi\|. \end{aligned} \tag{12}$$

One can therefore regard the amplitude A and phase θ of ϕ, instead of ϕ and $\phi^\dagger$, as two independent field degrees of freedom due to the vanishing equal-time commutation relation $[\mathrm{e}^{\mathrm{i}\theta(x)}, A(x')] = 0$. The situation is analogous to the definition of the two-mode phase operator [13, 14].

Within the quantized theory the charge of the CSF becomes an operator [8]:

$$Q = \mathrm{i}\int\mathrm{d}^3x : (\Pi^\dagger\phi^\dagger - \Pi\phi) := \sum_p(a_p^\dagger a_p - b_{-p}^\dagger b_{-p}) \tag{13}$$

where the prescription of normal ordering eliminates an infinite but unobservable vacuum charge. Note that [8]

$$[Q,\phi(x)] = -\phi(x) \qquad \big[Q,\phi^\dagger(x)\big] = \phi^\dagger(x) \tag{14}$$

we have $[Q, \phi^\dagger(x)\,\phi(x)] = 0$. Thus a question naturally arises: what is the appropriate representation that diagonalizes the charge-number operator Q and $\phi^\dagger(x)\,\phi(x)$ simultaneously?

Since a $\|\xi\rangle$ ($|\xi_p\rangle$) state does not have definite charge, the p-mode state of definite charge q_p can be obtained by the following prescription ($r_p > 0$):

$$|q_p, r_p\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi_p \sqrt{2r_p}\, \mathrm{e}^{-\mathrm{i}q_p\varphi_p} \left|\xi_p = r_p\, \mathrm{e}^{\mathrm{i}\varphi_p}\right\rangle$$
$$= 2\sqrt{\omega_p r_p} \sum_{n_p=\max(0,-q_p)} \mathrm{e}^{-\omega_p r_p^2 - a_p^\dagger b_{-p}^\dagger} \frac{(\sqrt{2\omega_p}\, r_p)^{2n_p+q_p}}{\sqrt{n_p!(n_p+q_p)!}} |n_p + q_p, n_p\rangle \tag{15}$$

where $|n_p + q_p, n_p\rangle \equiv [n_p!(n_p+q_p)!]^{-1/2}(a_p^\dagger)^{n_p+q_p}(b_{-p}^\dagger)^{n_p}|00\rangle_p$ and the integration is performed, using equation (7), over the $U(1)$-phase which is generated by the charge operator Q. It is a simple matter to show that

$$\begin{aligned}(a_p^\dagger a_p - b_{-p}^\dagger b_{-p})|q_p, r_p\rangle &= q_p|q_p, r_p\rangle\\ (a_p + b_{-p}^\dagger)(a_p^\dagger + b_{-p})|q_p, r_p\rangle &= 2\omega_p r_p^2 |q_p, r_p\rangle\end{aligned} \tag{16}$$

which means that the $|q_p, r_p\rangle$ states are the common eigenvectors of the p-mode charge operator $Q_p \equiv a_p^\dagger a_p - b_{-p}^\dagger b_{-p}$ and $(a_p + b_{-p}^\dagger)(a_p^\dagger + b_{-p})$. This is not surprising since the two operators are commutative. Remarkably, the $|q_p, r_p\rangle$ set spans a complete and orthonormal representation. A direct evaluation by using equation (11) yields

$$\begin{aligned}&\sum_{q_p=-\infty}^{\infty}\int_0^\infty \mathrm{d}r_p\, |q_p, r_p\rangle\langle q_p, r_p| = 1\\ &\langle q_p', r_p'|q_p, r_p\rangle = \delta_{q_p q_p'}\delta(r_p - r_p')\end{aligned} \tag{17}$$

which are the complete and orthonormal relations, respectively. This is in sharp contrast to the properties of the p-mode charge-conserved coherent states which are the common eigenstates of Q_p and $a_p b_{-p}$ [7, 15]. The charge-conserved coherent states with equal values of charge are, in general, not orthonormal and they are known to be overcomplete. In addition, we point out that $[a_p + b_{-p}^\dagger, a_p^\dagger + b_{-p}] = 0$. So we have the useful expressions $(a_p + b_{-p}^\dagger)|q_p, r_p\rangle = \sqrt{2\omega_p}\, r_p|q_p - 1, r_p\rangle$ and $(a_p^\dagger + b_{-p})|q_p, r_p\rangle = \sqrt{2\omega_p}\, r_p|q_p + 1, r_p\rangle$, which immediately lead to

$$\mathrm{e}^{\pm\mathrm{i}\theta_p}|q_p, r_p\rangle = |q_p \mp 1, r_p\rangle \tag{18}$$

with $\mathrm{e}^{\mathrm{i}\theta_p} \equiv (a_p + b_{-p}^\dagger)/[(a_p + b_{-p}^\dagger)(a_p^\dagger + b_{-p})]^{1/2} = \left(\mathrm{e}^{-\mathrm{i}\theta_p}\right)^\dagger$. Hence $\mathrm{e}^{\mathrm{i}\theta_p}$ ($\mathrm{e}^{-\mathrm{i}\theta_p}$) is the charge lowering (raising) operator for the p-mode.

It is straightforward to extend the above consideration to infinitely many degrees of freedom. Define $\|\{q\}, r\rangle \equiv \prod_p |q_p, r_p\rangle$, then from equations (1), (2), (13) and (15), we see that

$$Q\|\{q\}, r\rangle = \sum_p q_p\|\{q\}, r\rangle \tag{19}$$

and

$$\begin{aligned}\phi(x)\|\{q\}, r\rangle &= r(\boldsymbol{x})\|\{q-1\}r\rangle\\ \phi^\dagger(x)\|\{q\}, r\rangle &= r^*(\boldsymbol{x})\|\{q+1\}r\rangle\end{aligned} \tag{20}$$

where $r(\boldsymbol{x}) \equiv V^{-1/2} \sum_{\boldsymbol{p}} r_{\boldsymbol{p}}\, \mathrm{e}^{\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{x}}$. Equation (20) immediately gives

$$\phi^{\dagger}(x)\,\phi(x)\|\{q\}r\rangle = |r(\boldsymbol{x})|^2\|\{q\}, r\rangle. \tag{21}$$

The $\|\{q\}, r\rangle$ states are therefore the common eigenstates of Q and $\phi^{\dagger}(x)\,\phi(x)$. For the amplitude field operator defined in equation (12), $A(x)\|\{q\}, r\rangle = |r(\boldsymbol{x})|\|\{q\}, r\rangle$. Similar to equation (17), $\|\{q\}, r\rangle$ satisfy the functional complete and orthonormal relations:

$$\begin{aligned} &\sum_{\{q\}} \int [\mathrm{d}r]\|\{q\}, r\rangle\langle\{q\}, r\| = 1 \\ &\langle\{q'\}r'\|\{q\}, r\rangle = \delta_{\{q\}\{q'\}}\delta[r - r']. \end{aligned} \tag{22}$$

Thus the $\|\{q\}, r\rangle$ set spans another representation which can be called the *charge-amplitude representation*.

The advantage of the $\langle\{q\}, r\|$ representation is that within it, the charge lowering and raising operators can be defined very naturally. As can be seen from equations (12) and (20), we have

$$\mathrm{e}^{\pm\mathrm{i}\theta}\|\{q\}, r\rangle = \mathrm{e}^{\pm\mathrm{i}\arg r(\boldsymbol{x})}\|\{q \mp 1\}r\rangle \tag{23}$$

which mean that the operator $\mathrm{e}^{\mathrm{i}\theta}$ ($\mathrm{e}^{-\mathrm{i}\theta}$) reduces (increases) the charge of all modes of the CSF by one unit. The phase field operator $\mathrm{e}^{\mathrm{i}\theta}$ and its Hermitian conjugate $\mathrm{e}^{-\mathrm{i}\theta}$ thus serve, respectively, as the charge lowering and raising operators, regardless of the local phases $\mathrm{e}^{\pm\mathrm{i}\arg r(\boldsymbol{x})}$.

Importantly, it should be stressed that though our formulation is developed for the CSF, its applications are broader. Note that current EPR experiments rely on the two-particle entanglement [16], and the experimental realization of three-particle entanglement was reported only recently [17]. The entanglement involved in the field states opens a new avenue for the description of entanglement experiments with many-particle and field states. The fact that there exists the EPR entanglement for the free field states *in the original EPR sense* is striking. This remarkable fact shows the *precise* EPR correlation for the quantized fields at a fundamental level. The EPR entanglement involved in the field states, together with the recent demonstration of quantum non-locality for the original EPR states [18, 19], might imply quantum non-locality for the quantized fields as well.

Moreover, our formulation is applicable to the quantized electromagnetic field. To show this, it is convenient to use the Riemann–Silberstein complex vector [20] which can be quantized as

$$\boldsymbol{F}(\boldsymbol{x}, t) = \sum_{\boldsymbol{k}} \sqrt{\frac{|\boldsymbol{k}|}{V}}\boldsymbol{e}_{\boldsymbol{k}}\left[c_{\boldsymbol{k}}\,\mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x}-|\boldsymbol{k}|t)} + d_{\boldsymbol{k}}^{\dagger}\,\mathrm{e}^{-\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x}-|\boldsymbol{k}|t)}\right] \tag{24}$$

where $c_{\boldsymbol{k}}$ ($d_{\boldsymbol{k}}$) is the annihilation operator of the left-handed (right-handed) photons. Here the unit polarization vector $\boldsymbol{e}_{\boldsymbol{k}}$ satisfying $|\boldsymbol{e}_{\boldsymbol{k}}|^2 = 1$ and $\boldsymbol{k} \times \boldsymbol{e}_{\boldsymbol{k}} = -\mathrm{i}|\boldsymbol{k}|\boldsymbol{e}_{\boldsymbol{k}}$ is related to the polarization vector $\boldsymbol{e}_{\boldsymbol{k}}^{\pm}$ for left- and right-handed polarizations as $\boldsymbol{e}_{\boldsymbol{k}}^{+} = \boldsymbol{e}_{\boldsymbol{k}}$ and $\boldsymbol{e}_{\boldsymbol{k}}^{-} = \boldsymbol{e}_{\boldsymbol{k}}^{*}$. The close resemblance of equation (24) to equation (1) obviously shows the applicability of our formulation to the quantized electromagnetic field. Very recently, Braunstein and Kimble [21] proposed an elegant scheme for teleportation of continuous quantum variables by using squeezed-state entanglement whose limiting case can reduce to the original EPR entanglement. In fact, the field squeezed states can be naturally discussed in the $\langle\xi\|$ representation [10]. Combined with the Braunstein–Kimble scheme, the EPR states of quantum fields open up the new possibility for teleportation of many-particle quantum states with continuous spectra.

In conclusion, we have constructed the entangled eigenstates $\|\xi\rangle$ of complex scalar fields $\phi(x)$ and $\phi^{\dagger}(x)$. The $\|\xi\rangle$ states possess the complete and orthonormal properties and are a field-theoretical counterpart of the EPR pair states in quantum mechanics. From the $\|\xi\rangle$ states and within the prescription in equation (15), one can project out other complete and orthonormal states $\|\{q\}r\rangle$, the common eigenstates of the charge operator and $\phi^{\dagger}(x)\,\phi(x)$, with which the charge lowering and raising operators can be naturally defined. Thus we have provided two new representations (the $\langle\xi\|$ and $\langle\{q\},r\|$ representation), which may be useful in considering the representation and transformation theories of CSF. Thus our formulation is interesting in its own right. Moreover, the formulation is of conceptual importance in that it reveals the EPR correlation for the field states in the original EPR sense, and has broader applications, not merely in the CSF. The generalization of our formulation to the non-Abelian charged fields still remains a challenge in the near future.

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